A DEMONSTRATION OF TWO THEOREMS OF ELECTROSTATICS

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Denoting the electrical energy of a system of electrified points by W, we have

$$\begin{aligned} \mathrm{W} &= \sum \frac{q_m q_n}{r_{mn}} \\ &= \tfrac{1}{2} \Sigma \mathrm{V}_m q_m, \end{aligned}$$

q_m, q_n being the charges at the points m, n; r_{mn} the distance between these points; and V_m the potential at the point m. From this,

$$d\mathrm{W} = \tfrac{1}{2} \Sigma \mathrm{V}_m dq_m + \tfrac{1}{2} \Sigma q_m d\mathrm{V}_m. \qquad (1)$$

Consider the work $d\mathfrak{T}$ done by the electrical forces during a small displacement. We shall have

$$\begin{aligned} d\mathfrak{T} &= \sum \frac{q_m q_n}{r^2_{mn}} dr_{mn} \\ &= -\sum q_m q_n d\frac{1}{r_{mn}} \\ &= -d\sum \frac{q_m q_n}{r_{mn}} + \sum \frac{q_m dq_n - q_n dq_m}{r_{mn}} \\ &= -d\mathrm{W} + \Sigma \mathrm{V}_m dq_m. \end{aligned} \qquad (2)$$

By the aid of (1), this becomes

$$d\mathfrak{T} = d\mathrm{W} - \Sigma q_m d\mathrm{V}_m; \qquad (3)$$

and by adding (2) and (3) we get

$$d\mathfrak{T} = \tfrac{1}{2} \Sigma \mathrm{V}_m dq_m - \tfrac{1}{2} \Sigma q_m d\mathrm{V}_m. \qquad (4)$$

There are three cases of interest:

$$(1) \qquad dV_m = 0;$$
$$d\mathfrak{T} = dW. \quad \text{(Maxwell's Theorem)}$$

$$(2) \qquad dq_m = 0;$$
$$d\mathfrak{T} = -dW.$$

$$(3) \qquad d\mathfrak{T} = 0;$$
$$dW = \Sigma V_m dq_m = \Sigma q_m dV_m.$$

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